†KŠwYK MwZ m¤ú¦

f~wgKv

†Kvb we›`yi †Kvb A‡¶ Ave× GKwU e¯‘i Dci h‡_vchy³ ej c‡qvM Ki‡j, e¯‘wU A‡¶i Pvwi`‡K `yiZ AcwiewZZ †i‡L Nyi‡Z _v‡K, †Kbbv A‡¶ Ave× _vKvi `iæb e¯‘wU †mvRv c‡_ Pj‡Z cv‡i bv Aevi we›`yi A‡¶ Ave× bv _vK‡j I †K›`ªgyLx e‡ji Kvi‡Y Nyib m¤¢e n‡Z cv‡i| †hgb m‡hi Pvwi`‡K c„w_exi N~Yb A_ev wbDwKqv‡mi PZz`©‡K B‡j±‡bi AveZb| G NUbv¸wj‡K e„ËvKvi MwZ ev N~Yb MwZ e‡j| Mv‡gv‡dvb †iKW, Nwoi KvUvi MwZ, Mwoi PvKvi MwZ, ˆe`y¨wZK cvLvi MwZ cf„wZ. NUbvq N~Yb MwZi e¨e c‡qvM Avgiv †`L‡Z cvB|

cvV-1
†KŠwYK fi‡eM: N¥bi Z e¯‘z MwZkw³

D‡Ïk¨
G cvV †k‡l Avcwb-

- †KŠwYK fi‡e‡Mi GKK I gvÎv wjL‡Z cvi‡eb,
- RoZvi åvg‡Ki msÁv wjL‡Z cvi‡eb,
- N¥bi Z e¯‘z MwZi mgxKiY eYbv Ki‡Z cvi‡eb,
- PµMwZi e¨vmva Kv‡K e‡j wjL‡Z cvi‡eb|

5.1 : †KŠwYK fi‡eM

‰iwLK MwZi †¶‡Î Avgiv Rvwb, m f‡ii †Kvb e¯‘v ‡e‡M MwZkxj n‡j Zvi ‰iwLK fi‡eM **P**=m**v** | N¥bi Z †Kvb KYvi †¶‡Î †KŠwbK fi‡eM n‡"Q ˆiwLK fi‡e‡Mi Abyfic iwk| e‡Ëvi c‡_ †Kvb A‡¶i mv‡c‡¶ N¥qgvb KYvi ‰iwLK fi‡e‡Mi åvgKB n‡"Q KYvwUi †KŠwYK fi‡eM| myZivs e¯‘KYv e‡Ëvi c‡_ wbw`ó A‡¶i mv‡c‡¶ Nyi‡Z _vK‡j Zvi ˆiwLK fi‡eM I A¶ n‡Z ˆiwLK fi‡e‡Mi wµqv †iLvi j¤^`i‡Zi ¸Ydj‡K †KŠwYK fi‡eM e‡j|

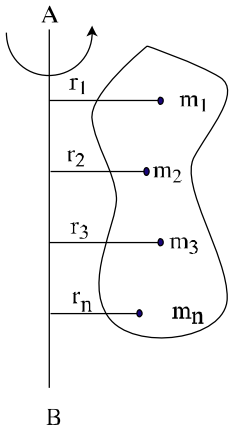
g‡b Kwi, M f‡ii GKwU e¯‘z m_1 , m_2 , m_3 --------- m_n . f‡ii n msL¨K KYv w`‡q ˆZix Ges A¶ n‡Z KYv¸wji `i‡Z h_vµ‡g r_1 , r_2 , r_3 , --------- r_n. (wPÎ 5.1) e¯‘wU e‡Ëi †K‡›`i Pviw`‡K ω mg-†KŠwYK †e‡M Nyi‡Z _vK‡j Gi cwZwU KYvi †KŠwYK †eM n‡e ω | wKšz‡K› `i n‡Z e¯‘KYv¸wji `i‡Z mgvb bv nIqvq G‡`i ˆiwLK †eM mgvb n‡e bv|

1g KYvi fi‡eM= $m_1 v_1 = m_1 r_1 \omega$

2q KYvi fi‡eM = $m_2 v_2 = m_2 r_2 \omega$

3q KYvi fi‡eM = $m_3 v_3 = m_3 r_3 \omega$

nZg KYvi fi‡eM = $m_n v_n = m_n r_n \omega$



wPÎ 5.1

1g KYvi †KŠwYK fi‡eM, $L_1 = m_1 r_1 \omega \times r_1 = m_1 r_1^2 \omega$

2q KYvi †KŠwYK fi‡eM, $L_2 = m_2 r_2 \omega \times r_2 = m_2 r_2^2 \omega$

3q KYvi †KŠwYK fi‡eM, $L_3 = m_3 r_3 \omega \times r_3 = m_3 r_3^2 \omega$

n Zg KYvi †KŠwYK fi‡eM, $L_n = m_n r_n \omega \times r_n = m_n r_n^2 \omega$

e¯‘wZi †KŠwYK fi‡eM KYv¸wj i †KŠwYK fi‡e‡Mi mgwói mgvb

awi, e¯‘wZi †KŠwYK fi‡eM = L

$\therefore L = L_1 + L_2 + ... + L_n$

$= m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega + ... + m_n r_n^2 \omega$

$= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ... + m_n r_n^2) \omega$

$= \Sigma m_i r_i^2) \omega$ GLv‡b i Øviv 1,2,3 -- BZ¨vw` msL¨v Ges Σ Øviv mgwói ewÂZvq ---------- (5-5)

$(\Sigma m_i r_i^2)$ †K AB A‡¶i mv‡c‡¶ e¯‘wZi RoZvi åvgK e‡j | G‡K I Øviv cKvk Kiv nq|

$\therefore I = \Sigma m_i r_i^2$

AZGe, wbw`ó A¶ mv‡c‡¶ N¥vqgvb e¯‘i c‡Z¨KwU KYvi fi Ges N¥vb A¶ n‡Z KYv¸wj `i‡Zi e‡Mi ¸Yd‡j i mgwó‡K H e¯‘i RoZvi åvgK e‡j |

M.K.S ev S.I c×wZ‡Z Gi GKK $kg.m^2$ Ges Gi gvÎv mL^2

Avevi mgxKiY (5-5) n‡Z cvB-

$L = \Sigma(m_i r_i^2) \omega$

ev, $L = I\omega$

mZ¨vs †KŠwYK fi‡eM = RoZvi åvgK × †KŠwYK †eM|

^iwLK MwZi †¶‡Î f‡ii †h fvjgKv †KŠwYK MwZi †¶‡Î RoZvi åvg‡Ki †m fvjgKv| mZ¨vs †Kvb e¯‘i †KŠwYK fi-†eM e¯‘wZi RoZvi åvgK I †KŠwYK †e‡Mi ¸bd‡j i mgvb|

5.2 : N¥vqgvb e¯‘i MwZkxj (Kinetic Energy of a Rotating Body):

awi, M f‡ii GKwU `p e¯‘ AB A‡¶i Pviw`‡K Ny‡Q| e¯‘wZ AB A‡¶i Pviw`‡K ω mg-†KŠwYK †e‡M Ny‡Z _vK‡j Gi cwZwU KYvi †KŠwYK †eM ω.

M f‡ii e¯‘wZ $m_1, m_2, m_3 ..... m_n$ f‡ii n msL¨K KYv wb‡q MwVZ Ges G KYv¸wj AB A¶ †_‡K `i‡Z h_vµ‡g $r_1, r_2, r_3 ..... r_i$ (KYG5.2)| †h‡nZzGB `i‡Z¸wj mgvb bq mZ¨vs KYv¸wj ^iwLK †eM mgvb n‡e bv|

AZGe,

m_1 fi we wkó KYv i ^i wLK †eM, $v_1 = r_1 \omega$

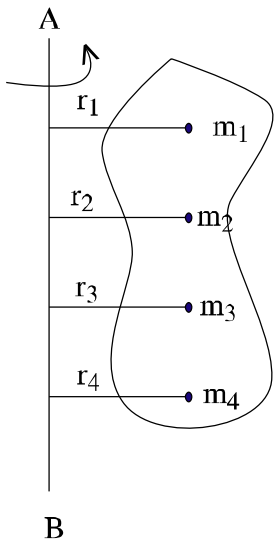
m_2 fi we wkó KYv i ^i wLK †eM, $v_2 = r_2 \omega$

m_3 fi we wkó KYv i ^i wLK †eM, $v_3 = r_3 \omega$

... ...

... ...

m_n fi we wkó KYv i ^i wLK †eM, $v_n = r_n \omega$



wP Î 5.2

Avevi, m_1 fi we wkó KYv i MwZ kw³, $E_1 = \frac{1}{2} m_1 v^2_1 = \frac{1}{2} m_1 r^2_1 \omega^2$

m_2 fi we wkó KYv i MwZ kw³, $E_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 r^2_2 \omega^2$

m_3 fi we wkó KYv i MwZ kw³, $E_3 = \frac{1}{2} m_3 v_3^2 = \frac{1}{2} m_3 r^2_3 \omega^2$

... ...

... ...

m_n fi we wkó KYv i MwZ kw³, $E_n = \frac{1}{2} m_n v_n^2 = \frac{1}{2} m_n r_n^2 \omega^2$

∴ mgM e¯'vi MwZ kw³, $E = E_1 + E_2 + E_3 + ... + E_n$

$= \frac{1}{2} m_1 r^2_1 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 + ... + \frac{1}{2} m_n r_n^2 \omega^2$

$= \frac{1}{2} \omega^2 [m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ... + m_n r_n^2]$

$= \frac{1}{2} \omega^2 \Sigma m_i r^2_i$ [GLv‡b Σ Øviv mgwó e¨vq|]

$= \frac{1}{2} I\omega^2$ ..... (5-6)

GLv‡b, $I = \Sigma m_i r^2_i = m_1 r^2_1 + m_2 r^2_2 + m_3 r^2_3 + ... + m_n r^2_n$; G‡K RoZv i åvgK e‡j |

msÁv t †Kvb NÈvbiZ `p e¯'zth AmsL¨ e¯'KYv i mgb‡q MwVZ, NÈvb A¶ †_‡K Zv‡`i c‡Z¨KwUi `i‡Zi eM I f‡ii ¸Yd‡j i mgwó‡K H NÈvb A‡¶i mv‡c‡¶ e¯'vZv i RoZv i åvgK e‡j |

D`vniY 3

5 wK‡jvMÖvg f‡ii PvKvi PµMwZi e¨vmva© 0.5 wgUvi| PvKwUi †KŠwYK †eM 31.4 †iwWqvb/†m:| Dnvi MwZkw³ wbY©q Ki“b|

mgvavb:

awi, RoZvi åvgK = I

Avgiv Rvwb,

$$I = mk^2$$

$= 5\times (0.5)^2$ wK‡jvMÖvg-wgUvi^2

$= 5\times 0.25$ wK‡jvMÖvg-wgUvi^2

$= 1.25$ wK‡jvMÖvg-wgUvi^2

GLv‡b,

m = 5 wK‡jvMÖvg

k = 0.5 wgUvi

Avevi

Avgiv cvB,

MwZkw³, $E = \frac{1}{2} I\omega^2$

$E = \frac{1}{2} \times 1.25\times (31.4)^2$ Ryj

$= 0.625\times 987$ Ryj

$= 616.87$ Ryj

GLv‡b,

$\omega = 31.4$ †iwWqvb/†m:

5.1.3 PµMwZi e¨vmva© (Radius of Gyration)

Avgiv Rvwb, RoZvi åvgK, $I = \Sigma m_i r^2_i$

GLb hw` mgM© e¯‘wUi fi M nq Ges e¯‘wUi m¤ú~Y© fi N~Y©‡bi A¶ †_‡K k `~i‡Z †K›`ªxf~Z Av‡Q e‡j aiv hvq Zvn‡j $I = \Sigma m_i r^2_i = Mk^2$

k †K PµMwZi e¨vmva© e‡j|

AZGe, N~Y©qgvb †Kvb `„p e¯‘i fi hw` GKwU wbw`©ó we›`y‡Z †K›`ªxf~Z e‡j aiv hvq hv‡Z GKwU wbw`©ó N~Y©b A¶ mv‡c‡¶ H †K›`ªxf~Z e¯‘KYvi RoZvi åvgK GKB A¶ mv‡c‡¶ mgM© `„p e¯‘i RoZvi åvgK‡Ki mgvb nq| Zvn‡j H wbw`©ó A¶ ‡_‡K we›`ywUi `~i‡K PµMwZi e¨vmva© e‡j| M.K.S ev S.I. c×wZ‡Z PµMwZi e¨vmv‡a©i GKK I ˆ`‡N¨©i GKK GKB|

mvims‡¶c

†KŠwYK †eMt e"e¯'vKvi c‡_ N~Ygvb †Kvb e¯'KYv GKK mg‡q e„‡Ëi †K‡›`ª †h †KvY Drcb K‡i Zv‡K e¯'z KYvwUi †KŠwYK †eM e‡j|

ˆiwLK †eM = †KŠwYK †eM × e„‡Ëi e¨vmva©|

RoZvi åvgK t wbw`ó A¶ mv‡c‡¶ N~Ygvb †Kvb e¯'i c‡Z¨KwU KYvi fi Ges N~Yb A¶ n‡Z KYv¸wji `~i‡Zi e‡Mi ¸Yd‡ji mgwó‡K H e¯'i RoZvi åvgK e‡j|

†KŠwYK fi‡eM = e¯'i †KŠwYK fi‡eM e¯'wUi RoZvi åvgK I †KŠwYK †e‡Mi ¸Yd‡ji mgvb|

c‡qvRbxq mgxKiY

1| *RoZvi åvgK t* $I = \Sigma m_i r_i^2$

2| *†KŠwYK fi‡e‡Mi gvb (e"vmq MwZi †¶‡Î) t* $L = mvr = mr^2\omega$

3| *†KŠwYK fi‡eM I RoZvi åvg‡Ki g‡a¨ m¤úK t* $L = I\omega$

4| *N~Yb MwZkw³ t* $E = \frac{1}{2} I\omega^2$

5| *RoZvi åvgK I PµMwZi e¨vmv‡ai g‡a¨ m¤úK t* $I = MK^2$

K. cv‡VvËi g~j¨vqb

mwVK DË‡ii cv‡k wUK wPý (√) w`b|

1. *†KŠwYK †e‡Mi mgxKiYwU wb‡Pi †KvbwU?*

   (K) *†KŠwYK †eM =* $\frac{\text{†KŠwYK `~iZ}}{\text{mgq}}$ (L) *†KŠwYK †eM = †KŠwYK `~iZ × mgq*

   (M) *†KŠwYK †eM =* $\frac{\text{‰iwLK †eM}}{\text{mgq}}$ (N) *†KŠwYK †eM =* $\frac{\text{mgq}}{\text{†KŠwYK `~iZ}}$

2. *ˆiwLK †e‡Mi mgxKiY wb‡Pi †KvbwU?*

   (K) *ˆiwLK †eM =* $\frac{\text{†KŠwYK †eM}}{\text{e„‡Ëi e¨vmva©}}$ (L) *ˆiwLK †eM = †KŠwYK †eM × e„‡Ëi e¨vmva*

   (M) *ˆiwLK †eM = e„‡Ëi e¨vmva × mgq* (N) *‰iwLK †eM =* $\frac{\text{†KŠwYK `~iZ}}{\text{e„‡Ëi e¨vmva}}$

3. M.K.S *c×wZ‡Z RoZvi åvg‡Ki GKK wb‡Pi †KvbwU?*

   (K) $N\ m^2$ (L) $kg\text{-}cm^2$

   (M) $gm.\ cm^2$ (N) $kg.\ m^2$

4. *ˆiwLK MwZi †¶‡Î f‡ii †h f~wgKv †KŠwbK MwZi †¶‡Î wb‡Pi †KvbwUi †m f~wgKv?*

   (K) *AvqZb* (L) *fi‡eM*

   (M) *RoZvi åvgK* (N) *IRb*

L. msw¶ß ck

1| msÁv wjLy

(K) †KŠwYK †eM, (L) †KŠwYK fi‡eM, (M) RoZvi åvgK, (N) PµMwZi e¨vmva|

2| †KŠwYK †e‡Mi iwkgvjv wbYq Ki“b|

3| ˆiwLK †eM I †KŠwYK †e‡Mi g‡a¨ m¤úK wK?

4| †KŠwYK †e‡Mi GKK wK?

5| †KŠwYK fi‡eM I RoZvi åvg‡Ki g‡a¨ m¤úK wjLy|

M. MvwYwZK ck

1| 4 wg. e¨vmva wewkó GKwU eË©vKvi c‡_ GKwU Mvox N›Uvq 50 wKtwgt †e‡M Pj‡Q| Gi †KŠwYK †eM wbYq Ki“b|

2| mh© I c„w_exi `~iZ $9.3 \times 10^7$ gvBj | c„w_ex m‚‡hi Pvwiw`‡K eË©vKvi c‡_ 365 w`‡b GKevi Nyi Av‡m a‡i wb‡q c„w_exi †KŠwYK I ˆiwLK †eM wbYq Ki“b|

3| GKwU nvZNwoi †m‡KÛ, wgwbU I N›Uvi KvUvi ˆ`N¨ h_vµ‡g .015m, .0125m I .03m | c‡Z¨‡Ki †kl cv‡šÍi ˆiwLK †eM wbYq Ki“b|

4| 10 †KwR f‡ii GKwU e¯‘ 4m ˆ`‡N¨i GKwU nvjKv f‡ii mZ©vi cv‡š† †e‡a Aci cv‡šÍi Pvwiw`‡K Nyiv‡bv n‡j e¯‘wUi RoZvi åvgK KZ n‡e?

cvV-2

UK ev †gvPo ev e¨eZb ej (Torque) I †KŠwYK MwZi Rb¨ wbDU‡bi m‡Ð

D‡Ïk¨

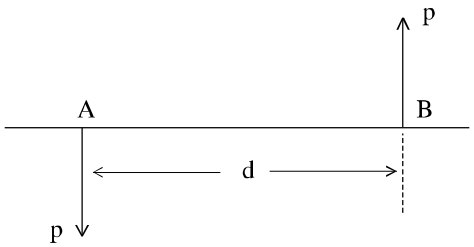
G cvV †k‡l Avcwb-

- Ø‡›`i msÁv wjL‡Z cvi‡eb|
- U‡Ki msÁv wjL‡Z cvi‡eb|
- †KŠwYK MwZi †¶‡Î wbDU‡bi MwZm‡Ð¸wj eYbv Ki‡Z cvi‡eb|

5.2.1. Ø›Ø ev Kvcj ev hyMj (Couple)

Ø›` t mgvb, mgvšivj I wecixZgyLx `yU ej hw` GKB we›`y‡Z wµqvi‡Z bv †_‡K wKQyv `i‡Z †i‡L wµqvi‡Z _v‡K, Z‡e G‡`i‡K Ø›Ø hv hyMj e‡j|

g‡b Kwi, †Kvb GKwU e¯‹z A I B we›`‡Z `¥wU mgvb, mgvšivj I wecixZgyLx ej wµqvi‡Z| msÁvbymv‡i, ej `yU Ø‡›`i mwó K‡i‡Q| Ø›Ø mwói d‡j e¯‹zUv mvg¨ve¯vq bv †_‡K N¥‡bi mwó K‡i| ejØ‡qi wµqv‡iLvi j¤-`i‡Z‡K Ø‡›Øi evû (arm of the couple) e‡j| wP‡Î AB n‡jv Ø‡›Øi evû (wPÎ-5-3)|



wPÎ 5.3

Avevi †h‡nZz†h †Kvb GKwU e‡ji gvb Ges ej `yUi ga¨eZx j¤ `i‡Zi ¸Ydj Øviv Ø‡›Øi åvgK cwigvc Kiv nq, myZivs Ø‡›Øi åvgK = P×AB = p×d .....(5-3)

Ø‡›Øi åvgK abvZK ev FYvZK n‡Z cv‡i| Ø‡›Øi wµqvi d‡j e¯‹zhw` Nwoi KvUvi wecixZ w`‡K N¥yi Z‡e Ø‡›Øi åvgK abvZK nq Avi hw` e¯‹zUv Nwoi KvUv †hw`K N¥yi †mw`‡K N¥yi Z‡e Ø‡›Øi åvgK FYvZK nq|

M.K.S ev S.I. c×wZ‡Z Ø‡›Øi åvg‡Ki GKK N-m Ges gvÎv ML^2T^{-2}|

5.2.2 : UK ev e‡ji åvgK (Moment of force or Torque)

†Kvb wbw`ó A‡¶i mv‡c‡¶ N¥yqvgvb †Kvb e¯‹zZ ZiY mwói Rb¨ Ø‡›Øi c‡qvRb Avi G Ø‡›Øi åvgK‡K UK e‡j| UK ej‡Z wbw`ó we›`y mv‡c‡¶ N¥biZ †Kvb e¯‹z Dci chy³ ej Ges H we›`yn‡Z e‡ji wµqv‡iLvi j¤ `i‡Z‡K eySvq|

e¨vL¨v I iwnkgvjv

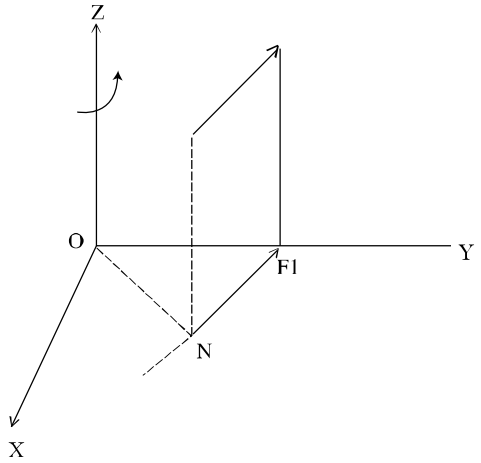
Avgiv Rwnb, †Kvb we›`yev A‡¶i mv‡c‡¶ Nÿvqgvb e¯‘i Dci UK c‡qvM Kiv nq| N¥bwe›`yA_ev A¶ n‡Z wKQy`ªæi ej c‡qvM KivB n‡"Q UK| UK‡K τ Øviv cKvk Kiv nq|

†Kvb A‡¶i mv‡c‡¶ †Kvb e‡ji åvgK ev UK ej‡Z ejwUi gvb I A¶ †_‡K e‡ji wµqv‡iLvi j¤ `i‡Zi ¸Ydj eSvq|

g‡b Kwi, †Kvb KYvi Nÿ‡bi A¶ OZ|
awi, XOY Z‡j †h †Kvb GKwU ej $\mathbf{F_1}$ wµqv Ki‡Q|
XOY Zj I OZ A‡¶i †Q` we›`yO n‡Z $\mathbf{F_1}$ e‡ji wµqv †iLvi Dci ON j¤ Awuk n‡jv| (wPÎ 5.4)
AZGe, OZ A‡¶i mv‡c‡¶ $\mathbf{F_1}$ e‡ji åvgK ev UK n‡e-

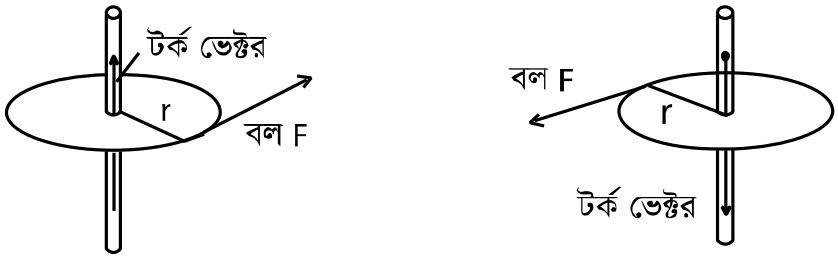
$\tau = \mathbf{F_1} \times \mathbf{ON} = \mathbf{F_1} \times \mathbf{d}$ ..... (5-4)

GLv‡b j¤ `i‡Z ON = d ‡K e‡ji evû (arm of the force) ej v nq|
myZivs UK = chy³ ej × e‡ji evû|



wPÎ : 5-4

e‡ji åvgK ev U‡Ki w`K: UK GKwU †fKUi iwnk| Gi GKwU wbw`ó Awfgy³ Av‡Q| UK †fKU‡ii Awfgy³ N¥bv A¶ eivei aiv nq e¯‘i AveZ‡bi w`‡K hw` GKwU `w¶Yv eZ©‡K Nÿwb nq| Z‡e ¯‹zU th w`‡K AMmi nq e‡ji åvgK ev UK †mw`‡K wµqv K‡i|



wPÎ 5.5

D`vniY 1

1 wgUvi e¨vm wewkó GKwU PvKvq Rov‡bv Zvi n‡Z 5 †KwR f‡ii GKwU e¯‘‡K w¯‘ifv‡e †Sv‡bv n‡jv| PvKvi A¶ mv‡c‡¶ UK wbYq Ki"b|

mgvavb:

Avgiv Rwnb,

UK τ = F×d

= 5×9.8×0.5

= 2.45 wbDUb-wgUvi

GLv‡b,

chy³ ej, F = 5×9.8×0.5 wbDUb

Ges e‡ji evû = $\frac{1}{2}$ wgUvi

= 0.5 wg.

N¥‡bi ceYZv

U‡Ki gvb †ewk n‡j N¥b †ewk nq| (5-4) bs mgxKiY n‡Z †`Lv hvq †h, åvgK evûi gvb Acwiewa _vK‡j A_vr d Acwiewa _vKv Ae¯vq N¥b wbfi Ki‡e e‡j gv‡bi Dci| GKBfv‡e e‡j gvb A_vr $F_1$ a"e _vKv Ae¯vq N¥b wbfi Ki‡e e‡j evûi Dci ev d Gi Dci (wPÎ 5-6) | e‡j wµqv‡iLv hw` N¥b A¶‡K †Q` K‡i Zvn‡j d kb¨ n‡e, G‡¶‡Î UK kb¨ n‡e| d‡j e¯‘wU Ny‡e bv| e‡j wµqv †iLv hw` Ny‡b A¶ A_vr OZGi mgvšivj nq, Zvn‡j N¥b A‡¶i j¤ w`‡K Aew¯Z Zj eivei Gi Dcvsk kb¨ n‡e| G †¶‡Î UK kb¨ n‡e A_vr e¯‘wU Ny‡e bv| myZivs U‡Ki gvb ej I N¥b evûi ga¨eZx †Kv‡Yi Dcil wbfi K‡i| UK e‡j j¤ eivei N¥b evûi Dcvsk ev Ny‡b evûi j¤ eivei e‡j Dcvs‡ki mgvbycwZK|

M.K.S ev S.I c×wZ‡Z U‡Ki GKK Newton-metre(N-m) ev Joule (J),

Ges gvÎv = $MLT^{-2} \times L = ML^2 T^{-2}$

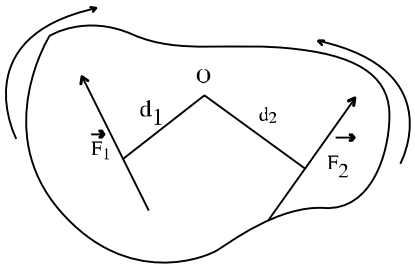
UK mg‡ni jwä

GKB mgq GKwU e¯‘z Dci `ßi ev Z‡ZwaK ej wµqv Ki‡j, e¯‘wU Ny‡e bv w¯i _vK‡e Zv H `ßi e‡j jwä UK Øviv wbawiZ nq|

awi, A GKwU e¯‘z(wPÎ: 5-6)| hvi Dci F_1 Ges F_2 ej wµqviZ| F_1 Ges F_2 ej c„Kfv‡e e¯‘z Dci wµqviZ _vK‡j e¯‘wU O Gi Pvi w`‡K Ny‡Z _vK‡e|

$\overrightarrow{F_2}$Gi cfv‡e Nwoi Kwdvi wecixZ w`‡K Ges $\overrightarrow{F_1}$ Gi cfv‡e Nwoi Kwdvi w`‡K N¥b msNwVZ n‡e|

∴ jwä UK = $F_2 d_2 - F_1 d_1$



wPÎ 5.6

hw`, $F_2 d_2 > F_1 d_1$ nq Z‡e e¯‘wU Nwoi Kwdvi wecixZ w`‡K Ny‡e| hw` $F_1 d_1 > F_2 d_2$ nq Z‡e e¯‘wU Nwoi Kwdvi w`‡K Ny‡e Ges hw` $F_2 d_2 = F_1 d_1$ nq Z‡e jwä UK kb¨ n‡e Ges e¯‘wU Ny‡e bv|

5.2.3: wbDU‡bi MwZm~Î

^iwLK MwZi g‡Zv †KŠwYK MwZi Rb¨I wbDU‡bi wZbwU m~Î i‡q‡Q| m~Î¸‡jv wb‡P e¨vL¨v Kiv n‡jv:-

c_g m~Ît ewni n‡Z UK c‡qvM bv Ki‡j, w¯'i e¯'z ¯'i _vK‡e Ges NyY©biZ e¯'zmg-†KŠwYK †e‡M Nyy©‡Z _vK‡e|

e¨vL¨vt G m~Î RoZvi ag©K e¨vL¨v K‡i| w¯'wZ RoZvi Rb¨ ewni n‡Z UK c‡qvM bv Ki‡j w¯'i e¯'z¯'i _v‡K| †hgb: `iRvi পাল্লায় UK c‡qvM bv Ki‡j †Lvjv `iRvi পাল্লা me mgq †Lvjv Ges eÜ `iRvi পাল্লা me mgq eÜ _v‡K|

GLb awi, †Kvb e¯'‡Z UK c‡qvM Kiv n‡jv| myÿvbyv‡i MwZ RoZvi Rb¨ NyY©biZ e¯'wU AbšKvj a‡i Nyy©‡e|

wØZxq m~Î : †Kvb e¯'i Dci wµqviZ UK Zvi †KŠwYK fi‡e‡Mi cwieZ©‡bi nv‡ii mgvbycvwZK Ges UK †h w`‡K wµqv K‡i †KŠwYK fi‡e‡Mi cwieZ©bI H w`‡K N‡U|

e¨vL¨v

Avgiv Rvwb

†KŠwYK fi‡eM $L = I\omega$

$\therefore$ †KŠwYK fi‡e‡Mi cwieZ©‡bi nvi $\frac{dL}{dt}$

m~Îvbyvqx,

$$\tau \propto \frac{dL}{dt},$$

ev, $\tau \propto I\frac{d\omega}{dt}$

ev, $\tau \propto I\alpha$ [GLv‡b $\frac{d\omega}{dt}$ †KŠwYK ZiY α]

ev, $\tau = kI\alpha$

GLv‡b k GKwU mgvbycvwZK a"eK| S.I GK‡K k=1

$\therefore \quad \tau = I\alpha$

A_©vr UK = RoZvi åvgK × †KŠwYK ZiY|

ZZxq m~Î: c‡Z¨K wµqvgy~K U‡Ki mgvb I wecixZ cwZwµqvgy~K UK Av‡Q|

e¨vL¨vt awi, A I B `ywU e¯'‡ A e¯'zB e¯'i Dci τ_A UK c‡qvM Ki‡j B e¯'z A e¯'i Dci mgvb I wecixZ τ_B UK c‡qvM Ki‡e|

m~Î Abyvqx, $\vec{\tau}_A = -\vec{\tau}_B$ .... (5-10)

mvims‡¶c

UK = †Kvb KYvi Dci chy³ ej Ges N¥b A¶ †_‡K e‡j i wµqv‡iLvi j¤ `ÿ‡Zi ¸Ydj‡K H A‡¶i mv‡c‡¶ UK e‡j|

$\tau = F \times d$

$\tau = I\alpha$

‰iwLK MwZi †¶‡Î ej c‡qvM e¨ZxZ e¯‘z ZiY mgó nq bv| GKBfv‡e UK chy³ bv n‡j †KŠwYK ZiY mgó nq bv|

cv‡VvËi gj¨vqb

mwVK DË‡ii cv‡k wUK wPý (√) w`b|

1. †KvbwU Øviv UK eySvq?
   (K) Fa (L) Iα
   (M) ma (N) Lα

2. †KŠwYK ZiY mgói Rb¨ wK c‡qvRb?
   (K) †KŠwYK †eM (L) RoZvi åvgK
   (M) UK (N) †KŠwYK MwZkw³|

msw¶ß DËi ck

1| UK ej‡Z wK eySvq?

2| UK I †KŠwYK Zi‡Yi g‡a¨ m¤úK wjLyb|

cvV-3
†K›`ªgyLx ej, hvbevnb I iv¯Ívi euvK|

D‡Ïk¨

GB cvV †k‡l Avcwb-

- ˆiwLK I †KŠwYK MwZi Zzjbv Ki‡Z cvi‡eb,
- †K›`ªgyLx e‡ji msÁv wjL‡Z cvi‡eb.
- †K›`ªgyLx e‡ji mgxKiY wjL‡Z cvi‡eb,
- eµc‡_ mvB‡Kj Av‡ivnxi MwZ eYbv Ki‡Z cvi‡eb,
- euvKi iv¯Ív wKsev †ij jvBb KvZ Kwiqv ivLv nq †Kb e¨vL¨v Ki‡Z cvi‡eb|

5.3.1. †KŠwYK fi‡e‡Mi msi¶Y

e¯‘i Dci UK c‡qvM Kiv n‡j Gi †KŠwYK fi‡e‡Mi cwieZb N‡U| wbDU‡bi wØZxq m~Îvbyhvqx-

$$\tau = \frac{dL}{dt} = I\frac{d\omega}{dt} \quad [\therefore L = I\omega,\ I \text{ a“e aiv n‡q‡Q}]$$

GLv‡b, L = †KŠwYK fi‡eM,

Avevi, hw` $\tau = 0$ nq, Z‡e-

$$\frac{dL}{dt} = 0$$

A_vr L = a“eK|

myZivs evwn¨K U‡Ki jwä kb¨ n‡j, N~Yvqgvb e¯‘i †KŠwYK fi‡e‡Mi †Kvb cwieZb nq bv| G‡K †KŠwYK fi‡e‡Mi msi¶Y m~Î e‡j|

**D`vniY:** mvKv‡mi mgq †L‡jvqvo hLb †`vjbv n‡Z Suc †`q ZLb nvZ I cv †mvRv cmwiZ K‡i| G mgq RoZvi åvgK I †ewk _v‡K Ges †KŠwYK †eM ω Kg _v‡K| †L‡jvqvoiwU nvZ I cv ¸wU‡q ey‡Ki Kv‡Q Avb‡j RoZvi åvgK I K‡g hvq Ges †KŠwYK †eM ω e„w× cvq| d‡j †m k‡b¨ K‡qKevi wWMevRx †L‡Z cv‡i| $I\omega$ a“e _v‡K e‡j Ggb N‡U|

5.3.2: ˆiwLK MwZ I N~Yb MwZi Zzjbv

wb‡P ˆiwLK I N~Yb MwZi Abyiƒc iwkgvjv cvkvcvwk †`Iqv n‡jv-

%iwLK	N~Yb
miY, s	†KŠwYK miY, θ
†eM, v	†KŠwYK †eM, ω
ZiY, f	†KŠwYK ZiY, α
fi, m	RoZvi åvgK, I
fi‡eM, mv, p	†KŠwYK fi‡eM, L
ej, F	UK, τ

D`vniY 1

1kg f‡ii GKwU e¯‘KYv‡K 0.60m `xN GKwU nvjKv iwk‡Z †eta cwZ †m‡K‡Û wbw`ó †K‡›`i Pwiw`‡K 5 evi Nyiv‡bv n‡jv| mgZvi Dci Uvb wbYq Ki|

mgvavb:

Avgiv Rwb,

$p = mrw^2$

$= 1\times.6\times (31.4)^2$

= 59.158 wbDUb (cvq)

GLv‡b,

fi, m = 1kg

e¨vmva, r = 0.6m

mgq, T = 1 †m:

K¤úvsK n = 5

∴ †KŠwYK †eM $\omega = 2\pi n$

$= 2\times3.14\times5$ †iwWqvb/†m:

= 31.4 †iwWqvb/†m:

D`vniY 2

3000 cvDÛ f‡ii GKwU †gvUi Mvox 1500 dzU e¨vmva‡ai GKwU eµ c‡_ N›Uvq 30 gvBj `"wZ‡Z Pj‡j †K›`wegyLx e‡ji gvb †ei Ki"b|

mgvavb:

Avgiv Rwb,

†K›`gyLx ej $F = \frac{mv^2}{r}$

$= 3000\times(44)^2$

= 3872 cvDÛvj

GLv‡b,

fi, m = 3000 cvDÊ

e¨vmva, r = 1500 dzU

`"wZ, v = 30 gvBj/N›Uv

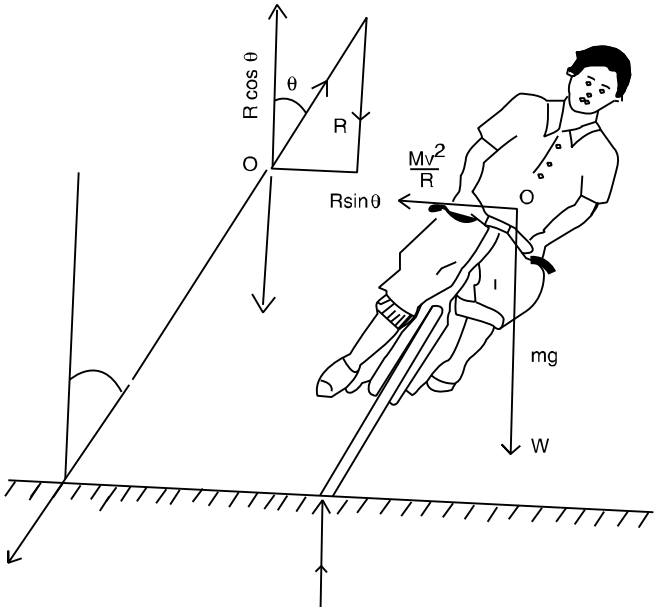
$= \frac{30\times1760\times32}{60\times60}$ dzU/†m.

= 44 dzU/†m:

5.3.4 : hvbevnb I i¯vi ewK

(1) eµc‡_ mvB‡Kj Av‡ivnxi MwZ (Motion of a cyclist along a curved path):

eµc‡_ Pjvi mgq mvB‡Kj Av‡ivnx mvB‡Kjmn Zvi kixi‡K †K‡›`i w`‡K †nwj‡q †`q| eµc‡_ Pjvi mgq Av‡ivnx I mvB‡K‡j †h †K›`wegvcx ej mwó nq Zv mvB‡Kjmn Av‡ivnx‡K wQU‡K †djvi †Póv K‡i| Kv‡RB G †K›`wegvcx ej‡K ckwgZ K‡i eµc‡_ MwZkxj _vKvi Rb¨ Av‡ivnx mvB‡Kjmn Zvi kixi‡K †K‡›`i w`‡K †nwj‡q †`q|



wPÎ 5.7

g‡b Kwi,

mvB‡Kj mn Av‡ivnxi fi = m
mvB‡Kj mn Av‡ivnxi IRb = mg
mvB‡Kj Av‡ivnxi `"wZ = v
ewKi e¨vmva = r

eËvKvi c‡_ Pj‡Z _vK‡j mvB‡Kj Av‡ivnxi †h †K›`gvcx e‡ji c‡qvRb nq Zvi gvb $\frac{mv^2}{r}$ | GB e‡ji †hvMvb w`‡Z mvB‡Kj Av‡ivnx উল্ল¤^i mwnZ θ †Kv‡Y †n‡j _v‡K|

GLv‡b mvB‡Kj mn Av‡ivnxi fvi †K›` n‡"Q O.

mvB‡K‡ji PvKv fvZgi Dci †h ej c‡qvM K‡i, fvZgI wecixZ w`‡K mgcwigvY cwZwµqv ej c‡qvM K‡i| g‡b Kwi GB cwZwµqv e‡ji gvb R|

R-Gi Aby‡fwgK Dcvsk = Rsinθ, GwU c‡qvRbxq †K›`gvcx e‡ji †hvMvb †`q|

R-Gi উল্লম্ব Dcvsk =Rcosθ, GwU IR‡bi mgZv i¶v K‡i|

$\therefore \quad R\sin\theta = \frac{mv^2}{r}$ Ges $R\cos\theta = mg$

AZGe, $\frac{R\sin\theta}{R\cos\theta} = \frac{\frac{mv^2}{r}}{mg}$ ev, $\tan\theta = \frac{v^2}{rg}$

$$\therefore\ \theta = \tan^{-1}\frac{v^2}{rg}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (5\text{-}14)$$

GB mgxKiY †_‡K †`Lv hvq, mvB‡Kj Av‡ivnxi ˆ`wLK †eM v eo n‡j ev e„ËvKvi c‡_i e¨vmva r †QvU n‡j θ eo nq|

mgxKiY 5-13 n‡Z eySv hvq-

(i) †ewk †e‡M †gvo wb‡Z n‡j, †ewk †n‡j _vK‡Z n‡e|

(ii) †ewk ev‡Ki iv¯—vq †ewk †n‡j _vK‡Z n‡e|

D`vniY 3

GKRb mvB‡Kj PvjK N›Uvq 15 gvBj `ªæZ‡Z PvjKvjxb 44 dzU e¨vmv‡ai GKwU †gv‡o euvK †bq| উল্ল‡¤i mv‡_ Zvi AvbZ †Kvb †ei Ki“b|

mgvavb:

Avgiv Rvwb,

$$\tan\theta = \frac{v^2}{rg}$$

$$= \frac{(22)^2}{44\times 32}$$

$$\therefore\ \theta = \tan^{-1} 0.344$$

GLv‡b,

`ªæZ, v = 15 gvBj/N›Uv

$= 15\times\frac{22}{15}$ dzU/†m:

= 22 dzU/†m:

e¨vmva, r = 44 dzU

(2) eµc‡_ Mvwoi MwZ (Motion of a car along a bend road):

moK c_ I †ijc_ A‡bK mgq euvKv _v‡K| Gme ev‡Ki iv¯—v wKsev †ij jvB‡bi eµZvi †K‡›`ªi cvk Aci cvk n‡Z wbPyivLv nq| KviY eµc‡_ Pjvi mgq †K›`ªvwZM e‡ji Afve nq, d‡j MwZ RoZvi cfv‡e Mvwo euvKvc‡Zi ¯úkK eivei †mvRv P‡j h‡Z Pvq hv Mvwo‡K eµZvi †K‡›`ªi wecixZ w`‡K wQU‡K †d‡j| euvKv iv¯—v ev †ijjvB‡bi eµZvi †K‡›`ªi cvk Aci cvk †_‡K wbPyivLvi e¨e¯—v‡K e¨vswKs e‡j|

g‡b Kwi,

PvjKmn Mvwoi fi = m

PvjKmn Mvwoi IRb = mg

Mvwoi `ªæZ = v

ev‡Ki e¨vmva = r

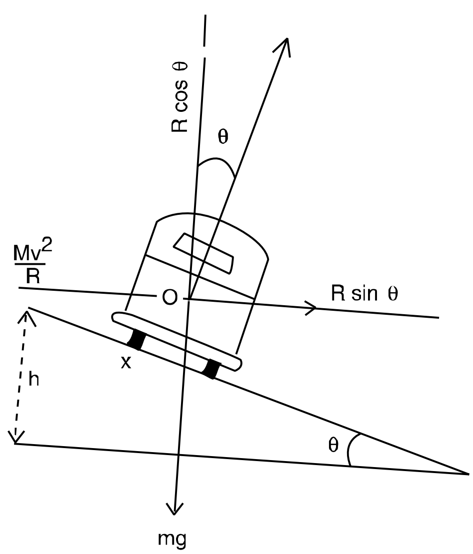
Mvwoi Dci iv¯—vi cwZwµqv ej = R

Ges iv¯—wU Abyf‚wg‡Ki mv‡_ eµZvi †K‡›`ªi w`‡K θ †Kvb ˆZix K‡i‡Q|

GLv‡b PvjKmn Mvwoi fvi‡K›` n‡”Q 0,

PvjKmn †nj‡bv mvB‡K‡ji g‡Zv Mvwoi Rb¨ †`Lv‡bv hvq †h-

$$\theta = \tan^{-1}\frac{v^2}{rg}\ \ldots\ldots\ldots\ldots\ldots\ (5\text{-}15)$$



wPÎ 5.8

Avevi iv¯v A_ev †ijjvB‡bi eµZvi †K‡›`i wecixZ cvk hw` †K‡›`i cvk A‡c¶v h D"PZvq _v‡K Ges iv¯v hw` x cwigvb PIov A_ev `yU jvB‡bi g‡a¨ `i-Z hw` x nq, Z‡e-

$$\sin\theta = \frac{h}{x} \quad \text{..... (5-16)}$$

$$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{h^2}{x^2}} = \sqrt{\frac{x^2-h^2}{x^2}}$$

$$\therefore \quad \tan\theta = \frac{h}{x}\sqrt{\frac{x^2}{x^2-h^2}}$$

$$\therefore \quad \tan\theta = \frac{h}{\sqrt{x^2-h^2}} \quad \text{..... (5-17)}$$

D`vniY

†ij jvB‡bi GKwU evuKi e¨vmva 100m Ges jvB‡bi `ßi cv‡Zi ga¨eZx` i-Z 1wgUvi| wfZ‡ii cvZ A‡c¶v ewn‡ii cvZ KZLwb DPyn‡j, ewn‡ii cv‡Z †Kvbiƒc Pvc c‡qvM bv K‡i GKwU †Ub 10 wgUvi/†m: `"wZ‡Z evuK wb‡Z cvi‡e? [g = 9.8 wgUvi/†m:^{2}]

mgvavb:

awi, wb‡Yq D"PZv = h

GLv‡b,

e¨vmva, r = 100 wgUvi

`"wZ, v = 10 wgUvi/†m:

Avgiv Rwb, $\tan\theta = \frac{v^2}{rg}$

$$= \frac{(10)^2}{100\times 9.8} = 0.1$$

θ Gi gvb ¶z`ª e‡j $\tan\theta = \sin\theta = \frac{h}{x}$ †jLv hvq|

$$\therefore \quad \tan\theta = \frac{h}{x} = 0.1$$

ev, $h = 0.1\times x$

$= 0.1\times 1 = 0.1$ wgUvi

mvims‡¶c

ewn¨K U‡Ki jwä kb¨ n‡j N¥vqgvb e¯‘i †KŠwYK fi‡e‡Mi †Kvb cwieZb nq bv|

eËvKvi c‡_ N¥vqgvb †Kvb e¯‘i Dci †h ej e¨Ëi ‡K‡›`i Awfgy³ wµqv K‡i e¯‘wU‡K eËvKvi c‡_ MwZkxj iv‡L Zv‡K †K›`gy³x ej e‡j|

c‡qvRbxq mgxKiY

1| †K›`gyLx ej: $F = \frac{mv^2}{r} = m\omega^2 r.$

2| iv¯vi evtjK mvB‡Kj Pvjbvi mgq Av‡ivnxi bwZ †KvY:

$\theta = \tan^{-1}\frac{v^2}{rg}$

3| Mwoi iv¯v †hLv‡b †gvo †bq †mLv‡b iv¯vi AvbwZ †KvY:

$\theta = \tan^{-1}\frac{v^2}{rg}$

cv‡VvËi gj¨vqb

mwVK DË‡ii cv‡k wUK wPý (√) w`b|

1. †K›`gyLx e‡ji mgxKiY wb‡Pi †KvbwU?

K) $F = m\omega^2 r$ L) $F = \frac{m\omega^2}{r}$

M) $F = \frac{\omega^2}{mr}$ N) $F = \frac{m}{r\omega^2}$

2. iv¯vi evtjKumvB‡Kj Pvjv‡bvi mgq Av‡ivnxi bwZ †Kv‡Yi gvb wb‡Pi †KvbwU?

K) $\theta = \sin^{-1}\frac{v^2}{rg}$ L) $\theta = \tan^{-1}\frac{v^2}{rg}$

M) $\theta = \cos^{-1}\frac{rg}{v^2}$ N) $\theta = \tan^{-1}\frac{rg}{v^2}$

msw¶ß ck

1| †KŠwYK fi‡e‡Mi wbZ¨Zvi mÿgZv eYbv I cgvY Ki“b|

2| †K›`gyLx ej Kv‡K e‡j? Gi gvb KZ? Awfj¤ ZiY Kv‡K e‡j?

3| evtjKi gyL iv¯v wKsev †ij jvBb KvZ K‡i ‰Zwi Kiv nq †Kb?

cvV-4
RoZvi åvgK

D‡Ïk¨

G cvV †k‡l Avcwb

- RoZvi åvg‡Ki Zvrch e¨vL¨v Ki‡Z cvi‡eb,
- RoZvi åvg‡Ki j¤ A¶mg‡ni Dccv`¨ cgvY Ki‡Z cvi‡eb,
- RoZvi åvg‡Ki mgvšivj A¶mg‡ni Dccv`¨ cgvY Ki‡Z cvi‡eb|

5.4.1 : RoZvi åvg‡Ki Zvrch

(1) ˆiwLK MwZi †¶‡Î Pjgvb e¯'i ˆiwLK fi‡eM = fi × †eM = Mv

NYb MwZi †¶‡Î e¯'i †KŠwYK fi‡eM = RoZvi åvgK × †KŠwYK †eM = Iω

(ii) ˆiwLK MwZi †¶‡Î e¯'i ˆiwLK MwZkw³ $= \frac{1}{2}$ × fi × (ˆiwLK †eM)^2 $= \frac{1}{2}mv^2$

NYb MwZi †¶‡Î e¯'i †KŠwYK MwZkw³ $= \frac{1}{2}$ × RoZvi åvgK × (†KŠwYK †eM)^2 $= \frac{1}{2}I\omega^2$

(iii) ‰iwLK MwZi †¶‡Î ˆiwLK Zi‡Yi Drm ej Ges ej = fi × ZiY = ma

†KŠwYK MwZi †¶‡Î †KŠwYK Zi‡Yi Drm UK Ges UK = RoZvi åvgK × †KŠwYK ZiY = Iα

myZivs ˆiwLK MwZi †¶‡Î e¯'i f‡ii †h f~wgKv, †KŠwYK MwZi †¶‡Î RoZvi åvg‡Ki †m f~wgKv| Avevi ˆiwLK MwZi †¶‡Î ej †hgb ZiY m„wó K‡i, †KŠwYK MwZi †¶‡Î UK †Zgwb †KŠwYK ZiY m„wó K‡i|

5.4.2: RoZvi åvg‡Ki Dccv`¨

RoZvi åvgK msµvšÍ `yÕwU Dccv`¨ i‡q‡Q: G¸wj n‡"Q-

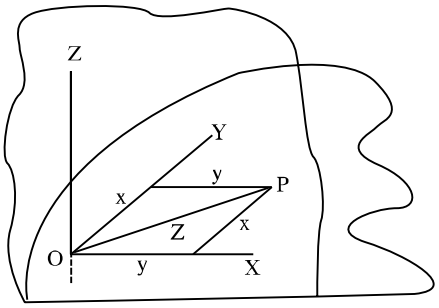
(K) j¤ A¶ mg‡ni Dccv`¨

(L) mgvšivj A¶ mg‡ni Dccv`¨

†Kvb GKwU A¶ m¤^‡c‡¶ †Kvb e¯'i RoZvi åvgK Rvbv _vK‡j GB Dccv`¨Ø‡qi mvnv‡h¨ mn‡R H A‡¶i mv‡_ j¤ ev mgvšivj Ab¨ †h †Kvb A¶ m¤^‡c‡¶ e¯'wUi RoZvi åvgK wbY©q Kiv m¤¢e|

**(K) j¤ A¶mg‡ni Dccv`¨** *t †Kvb mylg mgZj cv‡Zi Z‡j ci¯úi j¤f~‡e Ae¯Z `yB A‡¶i mv‡c‡¶ cvZ~Ui RoZv i åvgKØ‡qi mgwó A¶ `ywU i †Q` we›`y‡Z AskKZ j¤ A¶ mv‡c‡¶ cvZ~Ui RoZv i åvg‡Ki mgvb|*

awi, †Kvb mylg cv‡Zi mgZj ci¯úi j¤f~‡e Ae¯Z `yB A¶ OX Ges OY Gi mv‡c‡¶ cv‡Zi RoZv i åvgK h_vµ‡g* $I_X$ *I* $I_Y$*| H cv‡Z Ae¯Z Ges GB `yB A‡¶i †Q` we›`y‡Z AskKZ j¤ OZ *eivei cv‡Zi RoZv i åvgK* I_Z *n‡j* $I_X+I_Y= I_Z$



wP Î 5.9

cgvY *t mgZj cv‡Zi Dci* P *GKwU we›`y Ges* P *we›`y‡Z* m *f‡ii GKwU KYv Ae¯Z|* P *we›`y `i‡Z* OX *Ges* OY *n‡Z h_vµ‡g* x *Ges* y.

Avevi OZ *A¶ n‡Z* P *we›`y `i‡Z* z

GLb OZ *A¶ mv‡c‡¶ e¯‘Zv i RoZv i åvgK*

$I_Z = \Sigma mz^2$

$= \Sigma m(x^2+y^2), \quad [z^2 = x^2+y^2]$

$= \Sigma mx^2 + \Sigma my^2$ ..... (i)

wKšz $\Sigma mx^2 = I_X$ *Ges* $\Sigma my^2 = I_Y$

AZGe mgxKiY (i) *n‡Z cvB-*

$I_Z = I_X + I_Y$ ..... *(5-18)*

∴ *Dccv`¨wU cgwYZ nj|*

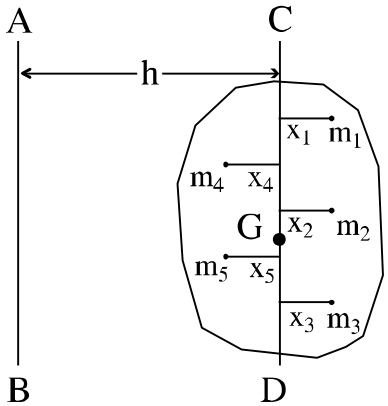
(L) mgvšivj A¶mg‡ni Dccv`¨t th †Kvb A¶ mv‡c‡¶ GKwU mgZj cv‡Zi RoZv i åvgK H A‡¶i mgvšivj Ges cv‡Zi fvi‡K›`Mvgx Aci GKwU A¶ mv‡c‡¶ cv‡Zi RoZv i åvgK Ges A¶Ø‡qi AšeZx `i‡Zi eM I cv‡Zi f‡ii ¸Ydj GB `yB‡qi mgwói mgvb|

awi, AB *A¶ mv‡c‡¶* M *f‡ii GKwU cvZjv mgZj cv‡Zi RoZv i åvgK* I.

CD *A¶* AB *A‡¶i mgvšivj Ges cv‡Zi fvi‡K›`Mvgx|*

Avevi CD *A¶ mv‡c‡¶ cvZ~Ui RoZv i åvgK* I_G *|* AB *I* CD *A¶Ø‡qi ga¨eZx `i‡Z* h*|*

cgvY Ki‡Z n‡e †h, $I = I_G + Mh^2$



wP Î 5.10

cgvb t awi, cvZwU m_1, m_2, m_3 ...... m_n BZ¨vw` AmsL¨ e¯‘KYvi mgb¦‡q MwVZ Ges CD A¶ †_‡K KYv¸wji `~iZ h_vµ‡g, x_1, x_2, x_3 .... x_n

$\therefore$ AB A‡¶i mv‡c‡¶ m_1 f‡ii KYvi RoZvi åvgK-

$= m_1(x_1+h)^2 = m_1 x_1^2 + m_1 h^2 + 2m_1 x_1 h$

GKB fv‡e AB A¶ mv‡c‡¶ m_2 f‡ii KYvi RoZvi åvgK

$= m_2^2 x_2^2 + m_2 h^2 + 2m_2 x_2 h$

$\therefore$ m_3 f‡ii KYvi RoZvi åvgK

$= m_3 x_3^2 + m_3 h^2 + 2m_3 x_3 h$ BZ¨vw` |

AB A‡¶i mv‡c‡¶ cvZwUi RoZvi åvgK I hv m_1, m_2, m_3 BZ¨vw` KYv¸wji RoZvi åvgK¸wji mgwói mgvb|

$\therefore$ $I = m_1x_1^2 + m_1h^2 + 2m_1x_1h + m_2x_2^2 + m_2h^2 + 2m_2x_2h + m_3x_3^2 + m_3 h^2 + 2m_3x_3h + ...$

$= \Sigma mx^2 + h^2 \Sigma m + 2h\Sigma mx$

GLv‡b Σmx, CD A¶ mv‡c‡¶ mgM cvZwUi f‡ii åvgK| wKšÍ†h‡nZz CD A¶wU fi‡K›`ªMvgx myZivs $\Sigma mx = 0$

Avevi $\Sigma m = M$ I $I_G = \Sigma mx^2$.

$\therefore$ $I = I_G + Mh^2$ .... (5-19)

Dccv`¨wU cgwYZ nj|

mvi ms‡¶c

ˆiwLK MwZi †¶‡Î e¯‘i f‡ii †h fygKv, †KŠwYK MwZi †¶‡Î RoZvi åvg‡Ki †m fygKv|

‰iwLK MwZi †¶‡Î ej ˆiwLK ZiY m„wó K‡i, GKBfv‡e †KŠwYK MwZi †¶‡Î UK †KŠwYK ZiY m„wó K‡i|

†Kvb my`„p mgZj cv‡Zi Z‡j ci¯úi j¤^fv‡e Aew¯'Z `ywU A‡¶i mv‡c‡¶ cvZwUi RoZvi åvgKØ‡qi mgwó H `yA‡¶i †Q`we›`y‡Z AswKZ j¤^ A¶ mv‡c‡¶ cvZwUi RoZvi åvg‡Ki mgvb|

†h †Kvb A¶ mv‡c‡¶ GKwU mgZj cv‡Zi RoZvi åvgK H A‡¶i mgvšÍivj Ges cv‡Zi fvi‡K›`ªMvgx Aci GKwU A¶ mv‡c‡¶ cv‡Zi RoZvi åvgK Ges A¶Ø‡qi AšeZ© `~i‡Zi eM© I cv‡Zi f‡ii ¸Yd‡ji mgwói mgvb|

c‡qvRbxq mgxKiY

1| j¤^ A¶mgv‡ni Dccv`¨ t $I_z = I_x + I_y$

2| mgvšÍivj A¶mgv‡ni Dccv`¨t $I = I_G + Mh^2$

D`vniY-4

GKwU cvZjv mylg eË©vKvi PvKwZi fi M I e¨vmva r, PvKwZi Z‡ji j¤ w`‡K Ges †K›`Mvgx GKwU A‡¶i mv‡c‡¶ PvKwZwUi RoZvi åvgK wbYq Ki“b|

mgvavb

aiv hvK, PQR GKwU PvKwZ| Gi †K›` O| XOY †iLvwU PvKwZi Dci j¤| PvKwZwU XOY A¶ mv‡c‡¶ NY©b¶g|

PvKwZwUi †¶Îdj = πr^2

$\therefore$ GKK †¶Îd‡ji fi = $\frac{M}{\pi r^2}$

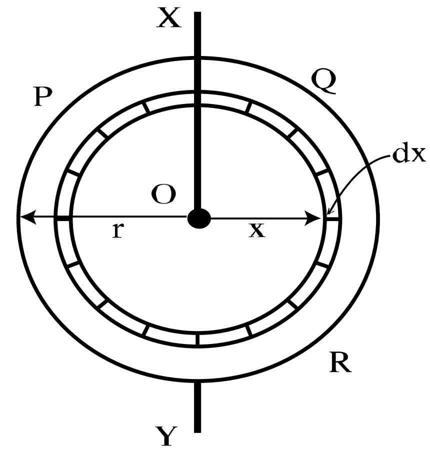
c_‡g, x e¨vmva wewkó PvKwZwUi GKwU ¶z` As‡ki åvgK wbYq Kiv hvK|

As‡kwUi we¯Ívi = dx

As‡kwUi †¶Îdj = $2\pi x.dx$

As‡kwUi fi = $2\pi x dx \times \frac{M}{\pi r^2}$

$= \frac{2M}{r^2}$ x.dx .... (1)



wPÎ 5.11

aiv hvK, XOY A‡¶i mv‡c‡¶ ¶z` As‡ki RoZvi åvgK = dI

$\therefore$ dI= fi × (`~iZ)^2= $\frac{2M}{r^2}$ xdx × x^2 = $\frac{2M}{r^2}$ x^3 dx ....(2)

(2) bs mgxKi‡Yi Dfq c¶‡K mgvKjb Ki‡j mgM PvKwZwUi RoZvi åvgK cvIqv hv‡e|

myZivs PvKwZwUi RoZvi åvgK,

$$I = \int dI = \int_0^r \frac{2M}{r^2} x^3\, dx$$ [$\because$ PvKwZwU x=0 Ges x=r mxgvi g‡a¨ we¯Í…Z]

$$= \frac{2M}{r^2} \int_0^r x^3\, dx = \frac{Mr^2}{2}$$

[Dt $\frac{Mr^2}{2}$]

D`vniY 5

GKwU cvZjv mylg eË©vKvi PvKwZi fi M I e¨vmva r| †h †Kvb GKwU e¨v‡mi mv‡c‡¶ PvKwZwUi RoZvi åvgK wbYq Ki“b|

mgvavb: aiv hvK, ADBC GKwU PvKwZ| AB Gi GKwU e¨vm|

PvKwZ AB mv‡c‡¶ NY©b¶g|

aiv hvK,

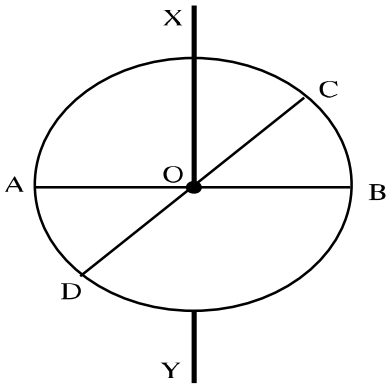
AB e¨vm Gi mv‡c‡¶ RoZvi åvgK = I ..... (1)

CD e¨vm Gi Ab¨ GKwU e¨v‡mi Ges CD⊥AB. PvKwZwU myiyg ZvB ‡h †Kvb e¨vm mv‡c‡¶ Gi RoZvi åvgK = I

∴ CD mv‡c‡¶ RoZvi åvgK = I.....(2)

O PvKwZwUi †K› `| XOY †iLvwU PvKwZwUi Dci j¤|

aiv hvK, XOY mv‡c‡¶ PvKwZwUi RoZvi åvgK = I/



wPÎ 5.16

Awfj¤ A¶ Dccv`¨ Abiyvqx,

$I^{/} = I+I$

ev, $2I = I^{/}$

ev, $I = \frac{I^{/}}{2} = \frac{Mr^2}{4}$ [∵ $I^{/} = \frac{Mr^2}{2}$, 4 bs D`vniY `óe¨]

DËi : $\frac{Mr^2}{4}$

K. cv‡VvËi gj¨vqb

mwVK DË‡ii cv‡k wUK wPý (√) w`b|

1. j¤ A¶mg‡ni Dccv‡`¨i mgxKiY wb‡Pi †KvbwU?

(K) $I_z = I_x + I_y$ (L) $I_z = \frac{Ix}{Iy}$

(M) $I = I_G + Mh^2$ (N) $I_z = I_x \times I_y$

2. mgvšivj A¶mg‡ni Dccv‡`¨i mgxKiY wb‡Pi †KvbwU?

(K) $I_z = I_x \times I_y$ (L) $I = I_G + Mh^2$

(M) $I = I_G \times Mh^2$ (N) $I_z = I_x + I_G$

L. msw¶ß ck

1| RoZvi åvg‡Ki Zvrch wjLyb|

2| j¤ A¶mg‡ni Dccv`¨wU wjLyb|

3| mgvšivj A¶mg‡ni Dccv`¨wU wjLyb|

iPbvgj¨K ck

1| (K) †KŠwYK †eM Kv‡K e‡j?

(L) †KŠwYK †e‡Mi iwkgvjv I w`K wbYq Ki“b|

(M) †KŠwYK †eM I ^iwLK †e‡Mi g‡a¨ m¤úK© cb Ki“b|

2| (K) †KŠwYK fi‡eM ev fi‡e‡Mi åvg‡Ki msÁv wjLyb

(L) RoZvi åvgK Kv‡K e‡j?

(M) GKwU w¯i A‡¶i Pvi w`‡K N¥vqgvb GKwU `p e¯‘z †KŠwYK fi‡e‡Mi Rb¨ iwkgvjv cwZcv`b Ki“b|

(N) N¥vqgvb e¯‘z MwZkw³i mgxKiY cwZcv`b K‡i RoZvi åvgK I PµMwZi e¨vmv‡ai g‡a¨ m¤úK© cb Ki“b|

3| wbDU‡bi MwZm~‡¸wj e¨vL¨v Ki“b|

4| (K) eËvKvi c‡_ mg-`“wZ‡Z Pjgvb †Kvb mvB‡Kj Av‡ivnx Zvnvi kixi e‡Ëi †K‡›`i w`‡K †nwj‡q †`q †Kb? e¨vL¨v Ki“b|

(L) evuKv iv¯vi e¨vswKs-Gi wK c‡qvRb? †gvUiMvwoi iv¯vi evuK c‡qvRbxq Avbwe‡Z †KvY wbYq Ki“b|

5| RoZvi åvgK msµvš Dccv`¨ `yBwee„Z I e¨vL¨v Ki“b|

6| †K›`gvjx ej Kv‡K e‡j| m f‡ii GKwU e¯‘zr e¨vmv‡ai GKwU eËvKvi c‡_ v mg`“wZ‡Z Nyi‡Q| cgvb Ki“b †h, $F = \frac{mv^2}{r}$ |